SIMULATION OF THE DIODE LIMITER USING LINEAR TIME VARIABLE FILTER

Jaromír Mačák
Doctoral Degree Programme (1), FEEC BUT
E-mail: xmacak04@stud.feec.vutbr.cz

Supervised by: Jiří Schimmel
E-mail: schimmel@feec.vutbr.cz

ABSTRACT

This paper deals with computer simulation of a diode limiter unit that can be found in many guitar distortion effect pedals. The algorithm is based on digital linear time-variable filter for simulation of behavior of a nonlinear circuit. Coefficients of the filter are changed in every sample period according to level of the input and output signal.

1 INTRODUCTION

Real-time digital simulation of analogue guitar effects plays important role in the field of the software for musicians and digital guitar effects have become very popular recently. However, implementation of these systems brings two contradictory requirements – accuracy versus computational complexity. Therefore the whole process of the simulation is divided into individual blocks and each block is simulated individually [1]. The analogue guitar distortion effects usually consist of filters and nonlinear blocks. The nonlinear block can be effectively implemented as a static waveshaper with good results [2]. Nevertheless, according to [3], a static nonlinearity doesn't work well on transients. A more accurate approach has been proposed in [3]. It is based on solution of nonlinear ordinary differential equations (ODE) using implicit (Backward Euler) and explicit (Forward Euler, Runga-Kutta) solvers. Linear time-invariant (LTI) filters can be considered as the solvers of the linear ODEs and they can be used for simulation of small-signal models of nonlinear systems because coefficients of these models are constant. However, in the guitar distortion effect, the large-signal models must be used. The large-signal models, which do not have constant parameters, can be described by a set of small-signal models with different parameters. So the linear time-variant filters must be used instead of the LTI filters.

2 DIODE LIMITER CIRCUIT MODEL

The diode limiter can be found in many guitar distortion effect pedals. The example of the diode limiter that provides one-way limiting is shown in Figure 1. This circuit consists of one signal source, resistor, capacitor and the nonlinear diode. The diode current is given by equation

$$I_{\mathrm{d}} = I_{\mathrm{s}}(\mathrm{e}^{\frac{U_{\mathrm{d}}}{U_{\mathrm{t}}}} - 1), \tag{1}$$

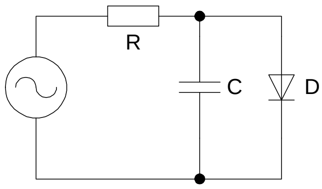


Figure 1: Diode limiter model.

where I_s is the saturation current, U_d is voltage on the diode and U_t is the thermal voltage. Using Kirchhoff's law we can obtain the nonlinear ODE

$$\frac{dU_d}{dt} = \frac{U_i - U_d}{RC} - \frac{I_s}{C}(e^{\frac{U_d}{U_t}} - 1). \tag{2}$$

This equation can be solved using some of the methods for numerical solving of the ODE mentioned in chapter 1.

2.1 SMALL-SIGNAL MODEL OF DIODE LIMITER

The diode from circuit in Figure 1 can be replaced with a nonlinear resistance. This leads to circuit in Figure 2

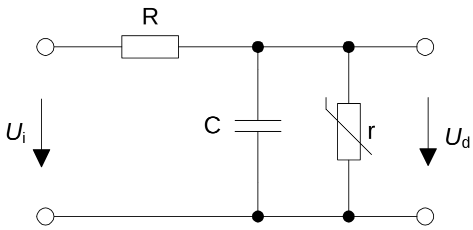


Figure 2: Small-signal model of diode limiter.

The nonlinear resistance r can be obtained from

$$r = \frac{dU_d}{dI_d} = \frac{U_t}{I_s e^{\frac{U_d}{U_t}}} \tag{3}$$

and it is considered as a constant in a small-signal model. Therefore it is possible to get transfer function of this circuit

$$S(p) = \frac{r}{r + R + pCRr}, \tag{4}$$

where p is the Laplace operator.
The bilinear transform [4]

$$p = 2f_s \frac{z-1}{z+1}, \tag{5}$$

where f_s is sampling frequency, of equation (4) results in

$$H(z) = \frac{r + rz^{-1}}{r + R + 2f_s CRr + (r + R - 2f_s CRr)z^{-1}} = \frac{a_0 + a_1 z^{-1}}{b_0 + b_1 z^{-1}}, \tag{6}$$

where a_0, a_1, b_0, b_1 are the LTI filter coefficients. The output signal is then

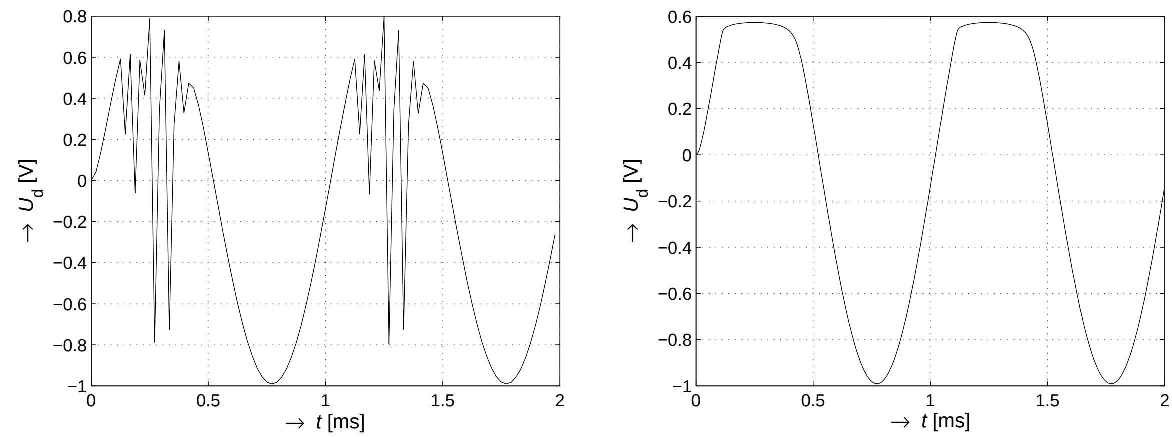
$$U_d[n] = \frac{a_0}{b_0} U_i[n] + \frac{a_1}{b_0} U_i[n-1] - \frac{b_1}{b_0} U_d[n-1]. \tag{7}$$

2.2 LARGE-SIGNAL MODEL OF DIODE LIMITER WITH LINEAR TIME VARIANT FILTER

A large-signal model works in a wider range of the input voltages than the small-signal model, so the nonlinear resistance r is now function of the output voltage U_{d}. The digital filter coefficients computed according to (6) are also functions of output voltage U_{d}. The output voltage equation (7) will change to

$$U_{\mathrm{d}}[n] = \frac{a_0(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])}U_{\mathrm{i}}[n] + \frac{a_1(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])}U_{\mathrm{i}}[n-1] - \frac{b_1(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])}U_{\mathrm{d}}[n-1]. \quad (8)$$

In this equation the output voltage value from the last iteration $U_{\mathrm{d}}[n-1]$ is used to compute filter coefficients $a_0(U_{\mathrm{d}}[n-1])$, $a_1(U_{\mathrm{d}}[n-1])$, $b_0(U_{\mathrm{d}}[n-1])$, $b_1(U_{\mathrm{d}}[n-1])$. Then the new output signal value is computed. Thus this algorithm works in iterative way in time like explicit methods for solving the ODEs. The output signal for 1 kHz sinewave input with amplitude of 1 V at sampling frequency of 48 kHz is shown in Figure 3. This algorithm needs to work at high sampling frequencies due to stability of the solution, see Figure 3a, where small sampling frequency is used. The 8-times oversampling has to be used for stability ensuring in this case (see Figure 3b). For higher signal values and higher signal frequencies it is necessary to use a higher oversampling (192-times oversampling for 10 kHz sinewave signal with amplitude of 10 V).



(a) No oversampling. (b) 8-times oversampling.

Figure 3: Output voltage for 1 kHz sinewave input with amplitude of 1 V.

The second possibility is exploiting the output signal value $U_{\mathrm{d}}[n]$ for computation of the filter coefficients. This algorithm is analogous to implicit methods for solving the ODEs. The output signal equation is in the following form

$$U_{\mathrm{d}}[n] = \frac{a_0(U_{\mathrm{d}}[n])}{b_0(U_{\mathrm{d}}[n])}U_{\mathrm{i}}[n] + \frac{a_1(U_{\mathrm{d}}[n])}{b_0(U_{\mathrm{d}}[n])}U_{\mathrm{i}}[n-1] - \frac{b_1(U_{\mathrm{d}}[n])}{b_0(U_{\mathrm{d}}[n])}U_{\mathrm{d}}[n-1]. \quad (9)$$

This equation has to be solved using the numerical methods. The nonlinear function

$$f(\mathbf{U}_{\mathrm{d}}[n],\mathbf{U}_{\mathrm{i}}[n]) = \frac{U_{\mathrm{i}}[n]+U_{\mathrm{i}}[n-1]}{\frac{I_{\mathrm{s}}}{U_{\mathrm{t}}}\mathrm{e}^{\frac{U_{\mathrm{d}}[n]}{U_{\mathrm{t}}}}} - \left(\frac{1-2f_{\mathrm{s}}CR}{\frac{I_{\mathrm{s}}}{U_{\mathrm{t}}}\mathrm{e}^{\frac{U_{\mathrm{d}}[n]}{U_{\mathrm{t}}}}}+R\right)U_{\mathrm{d}}[n-1] - \\ -\left(\frac{1+2f_{\mathrm{s}}CR}{\frac{I_{\mathrm{s}}}{U_{\mathrm{t}}}\mathrm{e}^{\frac{U_{\mathrm{d}}[n]}{U_{\mathrm{t}}}}}+R\right)U_{\mathrm{d}}[n] = 0, \tag{10}$$

where $\mathbf{U}_{\mathrm{d}}[n] = [U_{\mathrm{d}}[n], U_{\mathrm{d}}[n-1]]$ and $\mathbf{U}_{\mathrm{i}}[n] = [U_{\mathrm{i}}[n], U_{\mathrm{i}}[n-1]]$, is solved by the Newton method

$$U_{\mathrm{d}}^{k+1}[n] = U_{\mathrm{d}}^{k}[n] - \frac{f([U_{\mathrm{d}}^{\mathrm{k}}[n], U_{\mathrm{d}}[n-1]], \mathbf{U}_{\mathrm{i}}[n])}{f'([U_{\mathrm{d}}^{\mathrm{k}}[n], U_{\mathrm{d}}[n-1]], \mathbf{U}_{\mathrm{i}}[n])}, \tag{11}$$

where k is iteration index and value $U_{\mathrm{d}}^{0}[n]$ is the output signal estimation. The efficient estimator is filter with coefficients computed in the last iteration

$$U_{\mathrm{d}}^{0}[n] = \frac{a_0(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])}U_{\mathrm{i}}[n] + \frac{a_1(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])}U_{\mathrm{i}}[n-1] - \frac{b_1(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])}U_{\mathrm{d}}[n-1]. \tag{12}$$

The output signal for 10 kHz sinewave signal with amplitude of 10 V is shown in Figure 4a. The number of iterations required for each output signal sample is in Figure 4b. The estimated output signal value is same as the computed value in the linear part of the transfer function, so the number of iterations here is one. The bad estimation occurs in the nonlinear part of the transfer function (see Figure 4c) and the number of iterations rapidly grows here (dashed line in Figure 4b). This could be solved using saturation of estimation at value

$$U_{\mathrm{dmax}}^{0}[n] = f(U_{\mathrm{imax}}), \tag{13}$$

where

$$f(U_{\mathrm{i}}) = RI_{\mathrm{s}}(\mathrm{e}^{\frac{U_{\mathrm{d}}}{U_{\mathrm{t}}}}-1) + U_{\mathrm{d}} - U_{\mathrm{i}} \tag{14}$$

is circuit equation without capacitor. The saturation value can be obtained using the Newton method. Result of this improvement is shown in Figure 4d and the number of iterations has been reduced (solid line in figure 4b). The 8-times oversampling has been used to avoid aliasing that can cause problem with stability at higher frequencies.

3 CONCLUSION

Usage of the linear-time variant digital filters for simulation of the nonlinear dynamic system was discussed in this paper. The first type of the algorithm is simple in-time iteration. This algorithm has to work at very high sampling frequencies. The second type is based on the Newton method and it can work at relative low sampling frequencies. The efficiency of this algorithm depends on the estimation of the signal value in next sample period. The digital filter seems to be an efficient estimator, especially in the linear parts of the transfer function. In the nonlinear part of transfer function, saturation of the estimation was added to reduce the number of iterations. In future work a nonlinear circuit with a transistor will be solved using this method and we will deal with the estimator improvements.

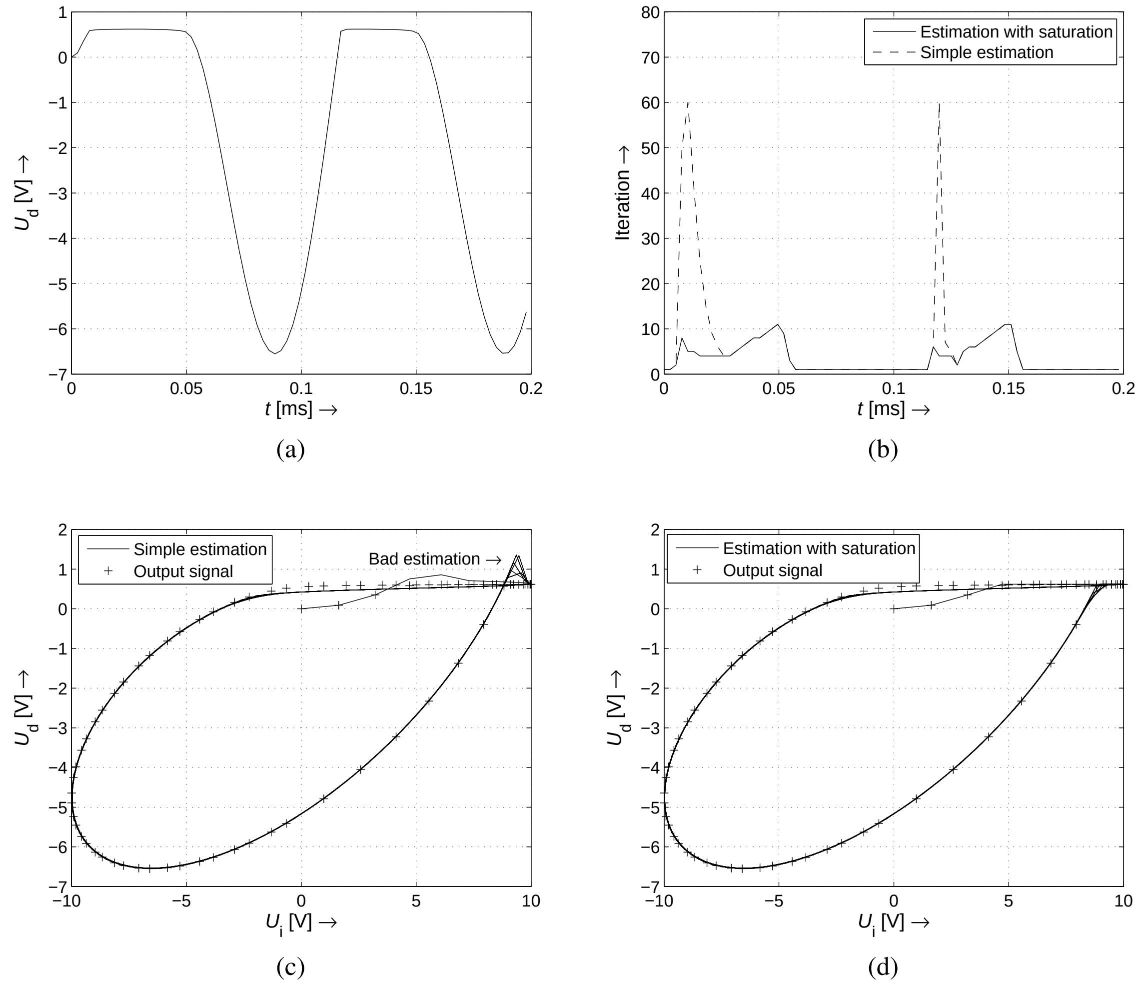


Figure 4: Behavior of the system for 10 kHz sinewave input with amplitude of 10 V. (a) Output voltage of the diode limiter. (b) Number of iterations per signal sample. (c) Dynamic transfer function of the diode limiter and simple filter used as the estimator. (d) Dynamic transfer function of the diode limiter and filter with saturation used as the estimator.

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